

Name _____ Test 2, Fall 2020

1) Find the determinant of the matrix below. (15 points)

$$\begin{bmatrix} 0 & 2 & 0 & 4 \\ 0 & 2 & 0 & -1 \\ 3 & 0 & 6 & 0 \\ 1 & -1 & 1 & 0 \end{bmatrix}$$

2) Given the basis and vector \vec{x}_B below, find \vec{x}_S . (10 points)

$$B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \end{bmatrix} \right\} \quad \vec{x}_B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

3) Given the two bases below, find the change of basis matrix that converts information from coordinate vectors in B_2 to coordinate vectors in B_1 , denoted by $[I]_{B_2}^{B_1}$. You do not need to perform the arithmetic. (10 points)

$$B_1 = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\} \quad B_2 = \left\{ \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} \right\}$$

4) Answer the questions below (3 points each)

(A) Let A be a 2×2 matrix with $|A| = 0$. How many solutions does $A\vec{x} = \vec{0}$ have?

(B) Let A be a 2×2 matrix with $|A| = 1$. What is the rank of A ?

(C) Let A be a 5×7 matrix with $\dim(NS(A)) = 3$. When row reduced, how many pivots does A have?

(D) Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^5$ be a one-to-one linear transformation with corresponding matrix A . When A is row reduced, how many rows of zeroes does it have?

(E) Suppose $A\vec{x} = \vec{b}$ is a system of equations that does not have a solution. If A is 4×5 , what is the minimum number of free variables in the system of equations.

5) Given the system of equations below, use Cramer's Rule to write down a formula for the solution. You do not need to simplify or evaluate your answer(s). (10 points)

$$3x + 2y = 5$$

$$4x - 6y = 7$$

The following row reduction may be useful for these problems.

$$\begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & -2 \\ 1 & 1 & 2 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$$

6) Are the vectors below linearly independent or linearly dependent? Justify your answer.

(10 points. 3 for the answer; 7 for the reasoning)

$$\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}, \text{ and } \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$$

7) Is the linear transformation below one-to-one? Justify your answer.

(10 points. 3 for the answer; 7 for the reasoning)

$$T(\vec{x}) = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & -2 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

8) Find the product below. (10 points)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 4 & 5 \\ -1 & 0 & 2 \end{bmatrix}$$

9) Find length of the vector below. (5 points)

$$\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

10) Given the system of equations below, identify which variable(s) are leading and which variable(s) are free. (5 points)

$$\begin{aligned} 2x + 3z &= 7 \\ 4y - 8z &= 6 \end{aligned}$$